

Heat transfer to a falling power-law fluid film

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Abstract

Steady heat transfer to water and purely viscous non-Newtonian fluid films falling down a vertical tube, for the case of uniform wall heat flux thermal boundary condition, was measured experimentally. The power-law fluids studied were aqueous solutions of Carbopol at 250 and 500 ppm (by weight) concentration. The heat transfer to power-law fluid film was lower than the Newtonian prediction, at comparable Pr_a . A new correlation is proposed for dimensionless heat transfer coefficient over a range of $500 \leq Re_a \leq 5400$, $6 \leq Pr_a \leq 20$, and $0.7 \leq n \leq 1$. © 1999 Elsevier Science Inc. All rights reserved.

Keywords: Falling film; Power-law fluid; Heat transfer

Notation

c_p	test fluid specific heat (J/(kg°C))
D	tube outside diameter (m)
g	acceleration due to gravity (m/s ²)
\bar{g}	$g(\rho - \rho_a)/\rho$ (m/s ²)
I	DC current through the tube (A)
h	local heat transfer coefficient $= \dot{q}''/(T_{wo} - T_b)$ (W/(m ² °C))
\bar{h}	peripherally averaged h (W/(m ² °C))
k	thermal conductivity of the test section wall (W/(m°C))
k_f	thermal conductivity of the fluid (W/(m°C))
K	consistency index of non-Newtonian fluid
L	tube length (m)
\dot{m}	mass flow rate $= \rho \dot{Q}$ (kg/s)
\dot{q}''	heat flux $= VI/(\pi DL)$ (W/m ²)
\dot{q}'''	volumetric heat generation rate $= VI/(\pi L(R_o^2 - R_i^2))$ (W/m ³)
\dot{q}_e	rate of electrical input $= VI$ (W)
\dot{q}_f	rate of enthalpy gained by the fluid $= \dot{m}c_p (T_{bo} - T_{bi})$ (W)
\dot{Q}	volumetric flow rate of fluid (m ³ /s)
R_i	inner radius of the test section (m)
R_o	outer radius of the test section (m)
t	film thickness (m)
T_b	fluid bulk temperature at any axial location (°C)
T_{bi}	fluid bulk temperature at the inlet (°C)
T_{bo}	fluid bulk temperature at the outlet (°C)
T_{bm}	mean bulk temperature $= (1/2)(T_{bi} + T_{bo})$ (°C)
T_f	film temperature $= (1/2)(T_b + T_{wo})$ (°C)
T_{wo}	outer wall temperature (°C)
\bar{T}_{wo}	peripherally averaged T_{wo} (°C)
T_{wi}	inner wall temperature (°C)

U	average velocity of the film $= \dot{Q}/[\pi(D+t)t]$ (m/s)
V	voltage drop across the tube length (V)
x	axial coordinate along the tube length (m)

Greek

τ	shear stress (Pa)
$\dot{\gamma}$	shear rate (s ⁻¹)
ρ	test fluid density (kg/m ³)
ρ_a	density of air surrounding the film (kg/m ³)
μ	Newtonian fluid viscosity (Pa s)
η	apparent viscosity of non-Newtonian fluid (Pa s)

Dimensionless quantities

h^*	dimensionless heat transfer coefficient, $(\bar{h}/k_f)(v^2/g)^{1/3}$
n	power law exponent
Pr	Prandtl number $= \mu c_p/k_f$
Pr_a	apparent Prandtl number $= \eta c_p/k_f$
Re	Reynolds number $= 4\dot{m}/(\pi D\mu)$
Re_a	apparent Reynolds number $= 4\dot{m}/(\pi D\eta)$
Re_f	film Reynolds number $= \rho Ut/\eta$
Y_i^+	$g^{1/2} t^{3/2}/\nu$

Suffixes

m	measured
p	predicted
w	at the test section wall

1. Introduction

Heat transfer to falling liquid films received much attention due to its applications in cooling systems of nuclear fuel clusters and in some chemical engineering operations. Analyses of turbulent heat transfer in falling liquid films using eddy diffusivity models have been developed for film evaporation and condensation (Dukler and Bergelin, 1952; Dukler, 1960;

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Mills and Chang, 1973; Kutateladze, 1982; Seban, 1954). Such phase-change processes rarely involve liquid films with Prandtl numbers greater than 10. Though heating and cooling of organic liquid films in chemical and food processing is often involved with high Prandtl number, such studies received much less attention (Kunz and Yerazunis, 1969).

The problem of stability of a liquid film has been the subject of many analytical investigations which addressed the film thickness, breakdown of thin films and subsequent re-wetting (Hartley and Murgatroyd, 1964; Hsu et al., 1963; Mikielewicz and Moszynski, 1976; Zuber and Staub, 1966). Norman and McIntyre (1960) measured minimum wetting rates for flow on the inside of a smooth pipe at high film inlet temperatures.

2. Non-newtonian fluids

The fluids encountered in process industries such as biochemical, petrochemical, food processing etc. are Non-Newtonian. Knowledge of the hydrodynamic and heat transfer behavior of these rheologically complex fluids is essential to an improved design of the equipment which handles such fluids.

An introduction to the classification of non-Newtonian fluids, and the constitutive equations for hydrodynamics of these fluids are provided by some excellent references (Bird et al., 1960; Middleman, 1968; Skelland, 1967; Wilkinson, 1960).

Non-Newtonian fluids exhibit non-linear relationship between shear stress (τ) and shear rate ($\dot{\gamma}$). The simplest mathematical model that describes the flow behavior of a non-Newtonian fluid is given by

$$\tau = K\dot{\gamma}^n, \quad (1)$$

where K and n are rheological constants for a given fluid. The apparent viscosity (η) of non-Newtonian fluids is a function of $\dot{\gamma}$:

$$\eta = \tau/\dot{\gamma}. \quad (2)$$

Several definitions of the Reynolds number have been used in the literature on non-Newtonian flows (Dodge and Metzner, 1959). One of the most commonly used is the apparent Reynolds number:

$$\text{Re}_a = 4\dot{m}/(\pi D\eta). \quad (3)$$

3. Experimental apparatus and procedure

The schematic of the flow loop (operated in a once-through mode) is shown in Fig. 1. Construction details of the apparatus, controls and instrumentation used can be found elsewhere (Rao, 1997). The test section is a polished stainless steel cylinder of 0.0254 m outside diameter and 0.33 m length. The cylinder was aligned vertical with a plumb line. At 11 axial locations spaced about 2.5 cm, copper-constantan (gage 30) thermocouples were mounted on the inner wall of the tube along isotherms. At each of those axial positions, 3 thermocouples were placed circumferentially (120° apart). In addition, two thermocouples were placed in the overhead tank and the mixing tank to measure the test fluid inlet and outlet bulk temperatures. The inner surface of the cylinder was insulated. The thermocouples were calibrated over the temperature range from 15°C to 40°C.

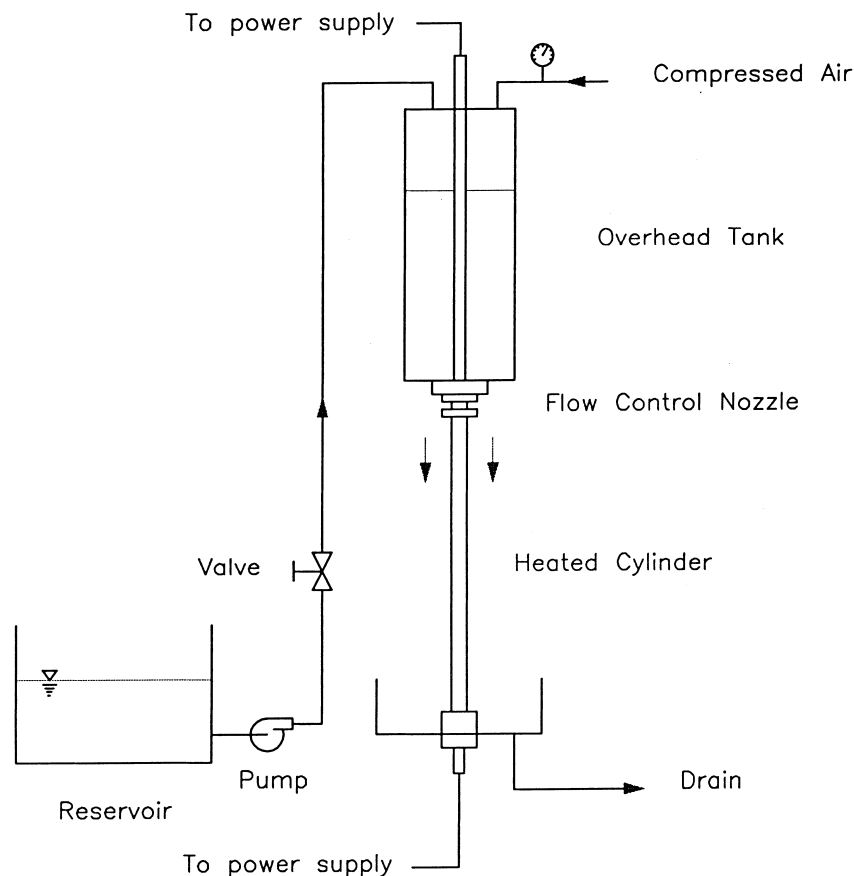


Fig. 1. Schematic of flow loop.

The test section was heated by passing direct electric current through, which yielded uniform heat flux thermal boundary condition at the outer wall. An *electrical-contact* probe was used to measure the time-averaged film thickness at only one axial location ($x/D \approx 6$) (Rao, 1997). However, the probe could be rotated about the test section longitudinal axis. A computer controlled mechanism was used to traverse the probe towards and away from the film to make and break the contact, respectively. The mass flow rate was deduced from the weight of the fluid (exiting the test section) collected during a measured interval of time.

The non-Newtonian test fluids, in this study, were aqueous solutions of Carbopol® 934 (a high molecular weight polymer powder) at concentrations 250 and 500 parts per million (by weight). In order to achieve lower values of n , the Carbopol solutions were neutralized with sodium hydroxide solution (10% strength) until an *optimal* PH value (≈ 7.2) was obtained. The rheology of polymer solutions is sensitive to solvent chemistry (Rao, 1997). Hence, for reproducibility of the results, distilled water was used as the solvent. The test section outer surface was cleaned and dried before wetting with a fresh batch of test fluid.

The test fluid was prepared in the reservoir at room temperature (21°C) and then pumped into the overhead tank (fitted with a liquid level indicator) from which the fluid was drained over the test section. The liquid level in the overhead tank was maintained constant by adjusting (automatically) the pump speed. To avoid dry patches, the liquid flow rate was set above the observed minimum wetting rate. The heat input was gradually increased until the temperature difference between the test section outer wall and the fluid bulk was about 5°C at $x/D \approx 10$.

The physical properties (except *viscosity*) of the aqueous polymer solutions were taken to be the same as for water, a Newtonian solvent (Lee et al., 1981). The steady shear viscosity of the polymer solutions at room temperature was measured using a Brookfield viscometer ($10^1 \text{ s}^{-1} < \dot{\gamma} < 10^3 \text{ s}^{-1}$), and a capillary viscometer ($10^3 \text{ s}^{-1} < \dot{\gamma} < 5 \times 10^4 \text{ s}^{-1}$). The results are shown in Fig. 2. A parabolic equation is fitted, in the range of shear rate measured in the experiment, and the rheological constants were evaluated:

$$\ln \tau = a_0 + a_1 \ln \dot{\gamma} + a_2 (\ln \dot{\gamma})^2 \quad (4)$$

$$n = a_1 + 2a_2 \ln \dot{\gamma}, \quad (5)$$

$$K = \exp[a_0 - a_2 (\ln \dot{\gamma})^2]. \quad (6)$$

At the test section outer wall, $\dot{\gamma}$ (required to estimate η) was unknown since the velocity profile across the film thickness was not measured. In light of this, for simplifying the present data analysis, the *reference* shear rate was taken as

$$\dot{\gamma} = U/t, \quad (7)$$

τ , η , and Re_a were calculated using Eqs. (4), (2) and (3) (at $\dot{\gamma}$ given by Eq. (7)), respectively. With the exception of mass flow rate and viscosity, all other measurements were made using a data acquisition system.

4. Results and discussion

The energy balance was monitored for all runs by comparing the electrical energy input (\dot{q}_e) with the rate of enthalpy gained by the test fluid (\dot{q}_f). \dot{q}_e was more than \dot{q}_f and the

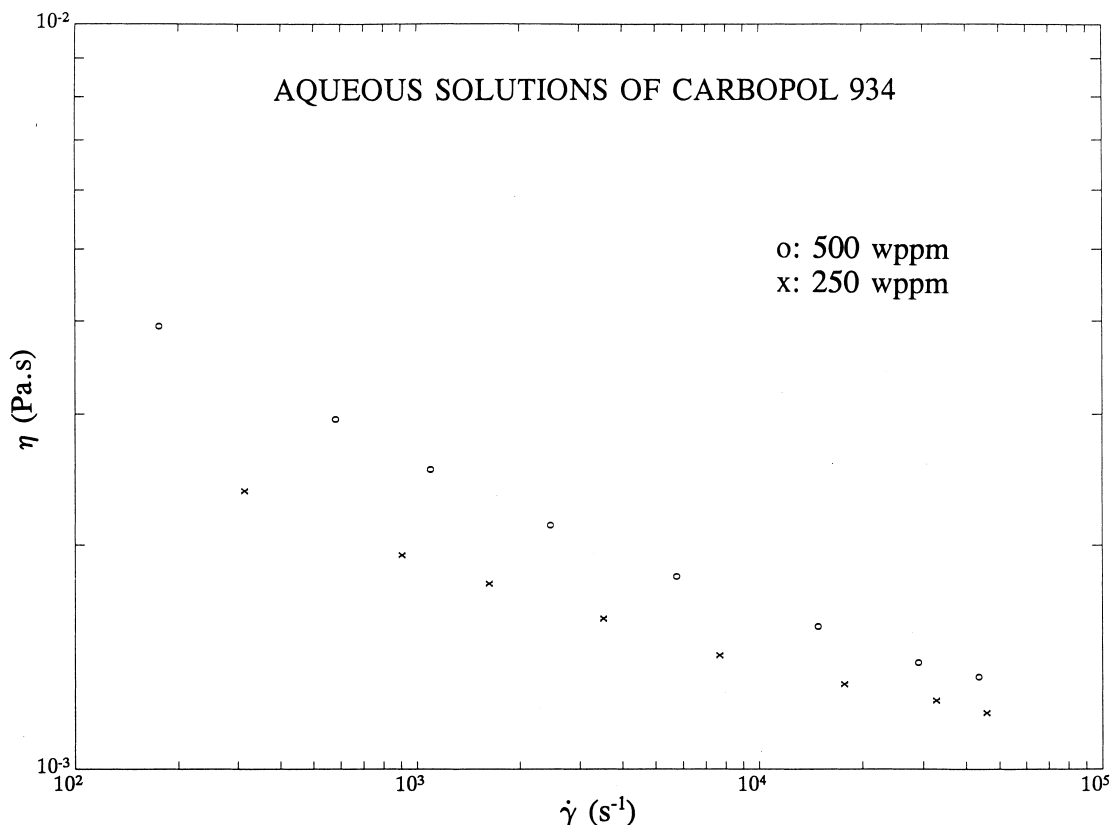


Fig. 2. Characteristic curves for aqueous polymer solutions.

deviation was less than 2.5% for all of the experimental runs reported here. Effects of radiative heat transfer and mass transfer due to evaporation of film were neglected. The estimated heat transfer rate from the film surface to the ambient air was less than 0.9% of \dot{q}_w'' .

The measured circumferential temperature distribution of the test section inner wall, at any given axial location, was within $\pm 1\%$ of the peripherally averaged value, suggesting that the one-dimensional radial conduction approach is reasonably accurate. Axial heat conduction in the test section wall was neglected.

The outer wall temperature was estimated, from the measured inner wall temperature, solving the one-dimensional steady state heat conduction equation:

$$T_{wo} = T_{wi} - (\dot{q}'''/4k)[R_o^2 - R_i^2 + 2R_i^2 \ln(R_i/R_o)]. \quad (8)$$

The bulk temperature of the fluid film at any distance downstream of the heated cylinder was calculated by applying an energy balance equation up to that point:

$$T_b(x) = T_{bi} + [VI(x/L)/\dot{m}c_p]. \quad (9)$$

The local heat transfer coefficient (h), at any x/D , was calculated at each angular position of the thermocouple at that x/D . At any given axial location, the dimensionless heat transfer coefficient (h^*) was, then, obtained from the peripherally averaged heat transfer coefficient (\bar{h}) at the same x . The test fluid thermal conductivity used to calculate the local h^* was evaluated at film temperature (T_f). No temperature correction was applied for viscosity since the wall to fluid bulk temperature difference was small. All other thermophysical properties of the fluid were taken at the mean bulk temperature (T_{bm}).

The minimum wetting rate (MWR) for isothermal trial runs of water, in this study, was $0.5 \times 10^{-5} \text{ m}^3/\text{s}$ which agreed with the semi-empirical Newtonian correlation (Munakata et al., 1975). For non-isothermal water runs reported here, the MWR (at $\dot{q}_w'' = 16.3 \text{ kW/m}^2$) was $0.3 \times 10^{-4} \text{ m}^3/\text{s}$ which is about 50% lower than the value reported by Hallett (1966). The MWR estimate is very sensitive to the contact angle (not measured in this study) based on the *force criterion* (Hartley and Murgatroyd, 1964). The film Reynolds number (Re_f) varied from 60 to 350 for water runs.

With increasing film flow rate and with increasing inlet fluid temperature, the heat flux to cause a dry patch becomes higher (Shires et al., 1964). Over a range of $1.3 \times 10^{-5} \text{ m}^3/\text{s} \leq \dot{Q} \leq 2.5 \times 10^{-4} \text{ m}^3/\text{s}$, and $0 \leq \dot{q}_w'' \leq 16.3 \text{ kW/m}^2$, no film breakdown was noticed in any run in this study. This observation is in agreement with the available experimental findings which suggest that the product of heat flux and the actual length of the distorted film to the dry patch (at breakdown) is sensibly constant at any given Reynolds number (Hsu et al., 1963). To avoid dry patches, the liquid flow rate was set above the observed MWR. The ranges of some of the parameters, in this study, are presented in Table 1.

Table 1
Ranges of parameters

	Water	Aqueous solutions of carbopol	
		250 wppm	500 wppm
Reynolds number	1037–5414	655–4440	502–4438
Prandtl number	5.9–6.1	14.6–14.9	19.1–19.9
n	1.0	0.82–0.84	0.70–0.72

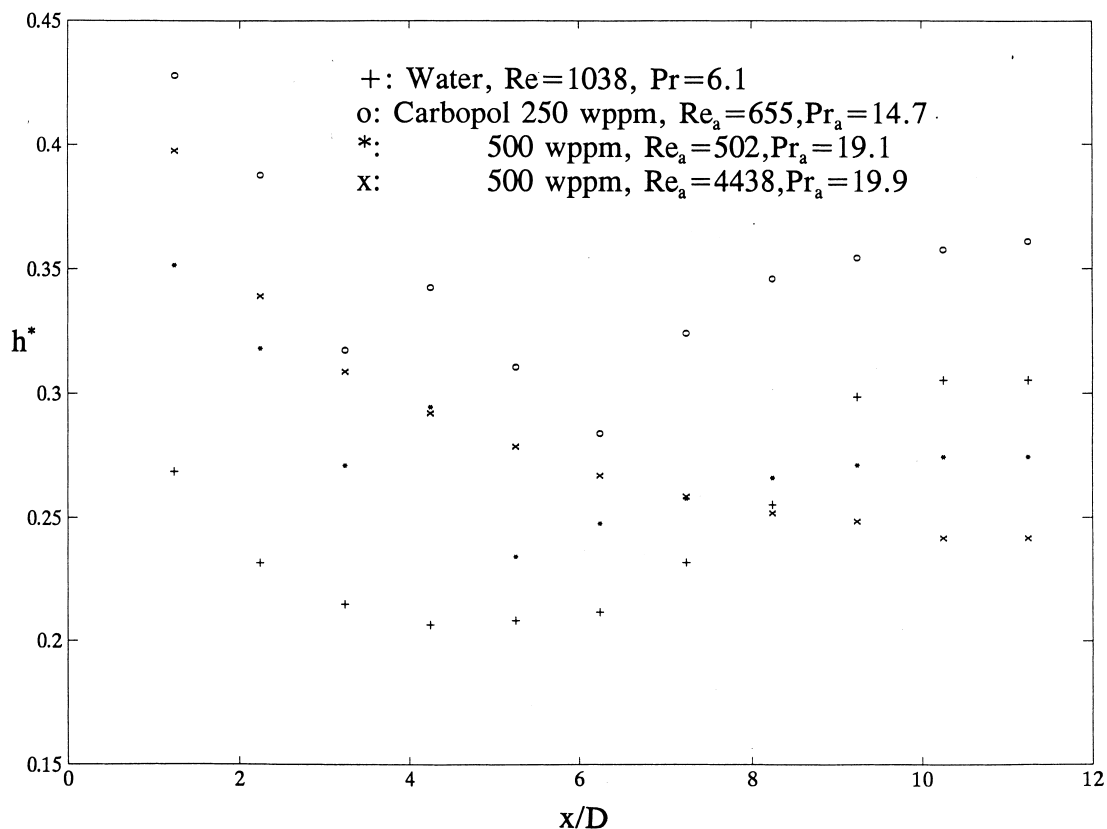


Fig. 3. Dimensionless heat transfer coefficient versus dimensionless axial distance.

The test fluid was drained through a special *flow-control* nozzle over the heated cylinder insuring that the flow was non-swirling and uniform around its circumference at the inlet. The maximum deviation of the measured film thickness (at three angular positions, 120° apart) from the circumferential average was less than 3% for all runs suggesting that the flow is nearly concentric at least up to $x/D \approx 6$. The measured time-averaged film thickness (at $x/D \approx 6$) for water runs reported here is within $\pm 5\%$ of the prediction (Fujita and Ueda, 1978):

Laminar film:

$$y_i^+ = [(3/4)\text{Re}]^{1/2}. \quad (10)$$

Turbulent film:

$$\text{Re} = -256 + 12y_i^+ + 10y_i^+ \ln y_i^+, \quad (11)$$

where y_i^+ is the dimensionless film thickness.

The measured h^* values for some typical experimental runs are shown as a function of x/D in Fig. 3. In the case of laminar water (a Newtonian fluid) film flow, the h^* decreased from a higher value near the test section entrance to a minimum (at $x_{\min}/D \approx 4$), then increased and gradually levelled off with x/D . The increase in h^* downstream of x_{\min} is due the boundary layer transition. The present data for water are in qualitative agreement with those in the literature (Oosthuizen and Cheung, 1977). The location where minimum h^* occurs, and the transition length (distance downstream from x_{\min} to where h^* levels off) depend on many parameters such as U, t, D etc. (Smith and Thompson, 1970). For laminar film flow of the aqueous polymer solutions studied here, h^* minima occurred at two locations ($x_{\min}/D \approx 2.5$ and 5) on either side of the observed for water. In turbulent flow, however, no local minima were noticed. No other results are available in the literature on x_{\min} for power-law fluid films.

No studies are published on the thermal developing length for falling non-Newtonian liquid films. However, the heated cylinder in this study was long enough to obtain the fully developed h^* values for water as well as power-law fluid films. This is evident from Fig. 3 as well as the measured wall temperature distribution (not shown here).

The measured fully developed h^* values for water ($\text{Pr} \approx 6$) film are shown in Figs. 4 and 5. For a laminar fully developed Newtonian falling film with uniform wall heat flux (Ueda and Tanaka, 1974):

$$h^* = 2.27 \text{Re}^{-1/3}. \quad (12)$$

Eq. (12) is applicable when all of the heat from the wall is absorbed in the liquid film. For turbulent heat transfer to falling liquid film (Brauer, 1956; Wilke, 1962):

$$h^* = 0.0323 \text{Re}^{1/5} \text{Pr}^{0.344}, \quad 2460 \text{Pr}^{-0.646} \leq \text{Re} < 1600, \quad (13)$$

$$h^* = 0.00102 \text{Re}^{2/3} \text{Pr}^{0.344}, \quad 1600 \leq \text{Re} \leq 3200, \quad (14)$$

$$h^* = 0.00871 \text{Re}^{2/5} \text{Pr}^{0.3444}, \quad 3200 \leq \text{Re}. \quad (15)$$

From Figs. 4 and 5, it can be seen that the present *fully developed* h^* values for water in the laminar region are slightly underpredicted by Eq. (12) whilst those in the turbulent region are overpredicted by (Eqs. (13)–(16)) for $\text{Pr} \approx 6$. Non-uniform film formation decreases the h^* to the values smaller than those yielded by Eqs. (13)–(15) (Ponter and Davies, 1968). In addition, the measured h^* depends on the test section geometry, alignment, and surface roughness, velocity distribution at the entrance, $T_{\text{bi}}, \dot{q}_w'', \text{Re}_a, \text{Pr}_a$ etc. Film thinning (visually observed in this study for $x/D > 8$) caused the present turbulent h^* data to be lower than the prediction above (Eqs. (12)–(15)).

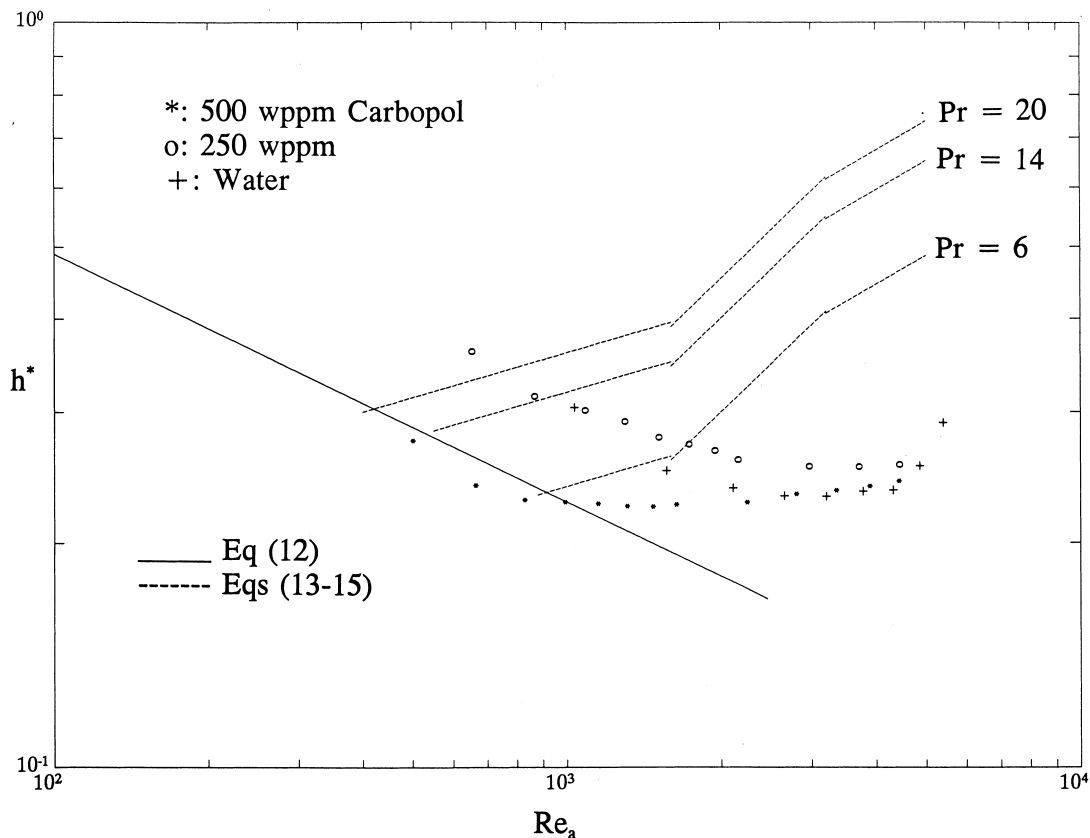


Fig. 4. Fully developed dimensionless heat transfer coefficient versus apparent Reynolds number.

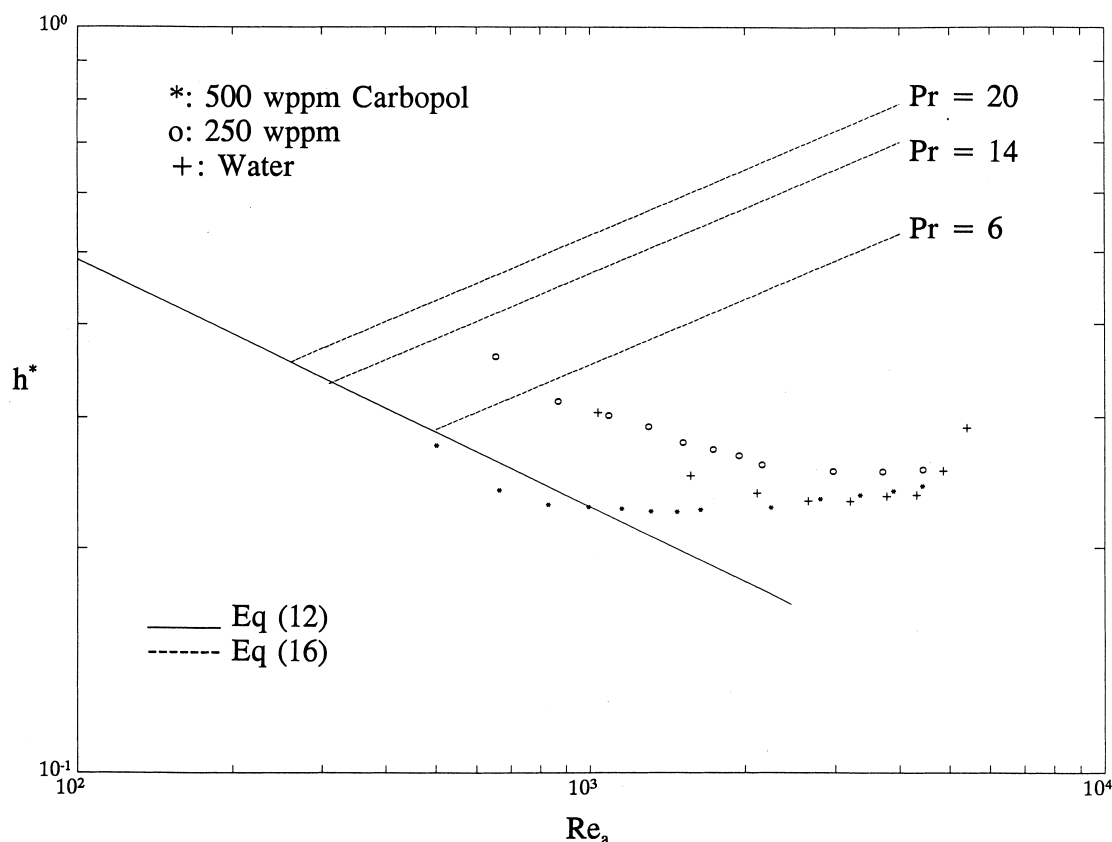


Fig. 5. Fully developed dimensionless heat transfer coefficient versus apparent Reynolds number.

The fully developed h^* for the aqueous Carbopol solutions are shown in Figs. 4 and 5, from which it can be noticed that laminar to turbulent transition occurs at lower Re_a as the Pr_a increases. This observation is consistent with the available Newtonian results (Wilke, 1962). Also shown in Fig. 5 is the correlation proposed by Carey (1985), based on an approximate analysis, for turbulent heat transfer to falling Newtonian liquid films at high Prandtl number:

$$(h/k_f)(\nu^2/g)^{1/3} = 0.0259 Pr^{1/3} Re^{7/24}, \quad 5 \leq Pr \leq 210. \quad (16)$$

The Pr_a for the Carbopol solutions of 250 and 500 wppm in this study are about 14 and 19, respectively. Both Figs. 4 and 5 show that the present limited experimental fully developed h^* values for falling power-law fluid films are underpredicted in the laminar region and overpredicted in the turbulent region by the Newtonian correlations (Eqs. (12)–(16)), at comparable Pr_a . From Figs. 4 and 5 it can also be seen that, at any given Re_a , the turbulent h^* reduction (difference between the predicted and the measured) increases with decreasing value of n . Apparently, the effect of higher Pr_a (i.e. with increase in polymer concentration) to increase h^* is outweighed by the influence of lower power-law index to decrease h^* .

The experimental conditions and measurement uncertainties cited in the literature varied considerably. For example, Wilke's data were obtained from a vertical tube heated by hot water flowing through inside of the tube (Wilke, 1962). Oosthuizen and Cheung (1977) reported turbulent h^* for developing water film flow. Fujita and Ueda (1978) used an alternating electric current as the heat source and the film flow fully developed upstream of the heated section. Their turbulent h^* values are also overpredicted, at q''_w comparable to the value

in this study, by Eqs. (13)–(15). Some investigators failed to mention the errors in temperature measurement if the thermocouples were not laid out along the isotherms. A few presented heat transfer coefficients averaged over the test section length.

The relative decrement (difference between the measured and the predicted) in the present fully developed turbulent h^* (as shown in Fig. 4) is attributed mainly to the non-Newtonian nature of the fluid and partly to the experimental conditions here.

No analytical or other experimental results for power-law fluid falling films, with which to compare the present data, are available. More experimental data are needed to verify if the turbulent h^* reaches an asymptotic value as n approaches zero. Quantitative information on the surface tension of aqueous polymer (at different concentration) solutions and its effect on film thinning helps in understanding the heat transfer characteristics of falling films of power-law fluids.

The aqueous solutions of Carbopol become quite viscous with increasing polymer concentration. The (positive displacement) pump capacity set the limit for the highest concentration and the flow rate (500 wppm, and $2.5 \times 10^{-4} \text{ m}^3/\text{s}$, respectively) used in this study.

Turbulent heat transfer to purely viscous power-law fluid flows is often correlated explicitly in terms of n (Rao, 1994). Based on the present limited experimental data, the following new correlation is proposed for fully developed h^* for falling power-law liquid films

$$h^* = 0.009 Re^{0.32} Pr^{1/3} f(n), \quad (17)$$

$$f(n) = A + Bn + Cn^2 \quad (17a)$$

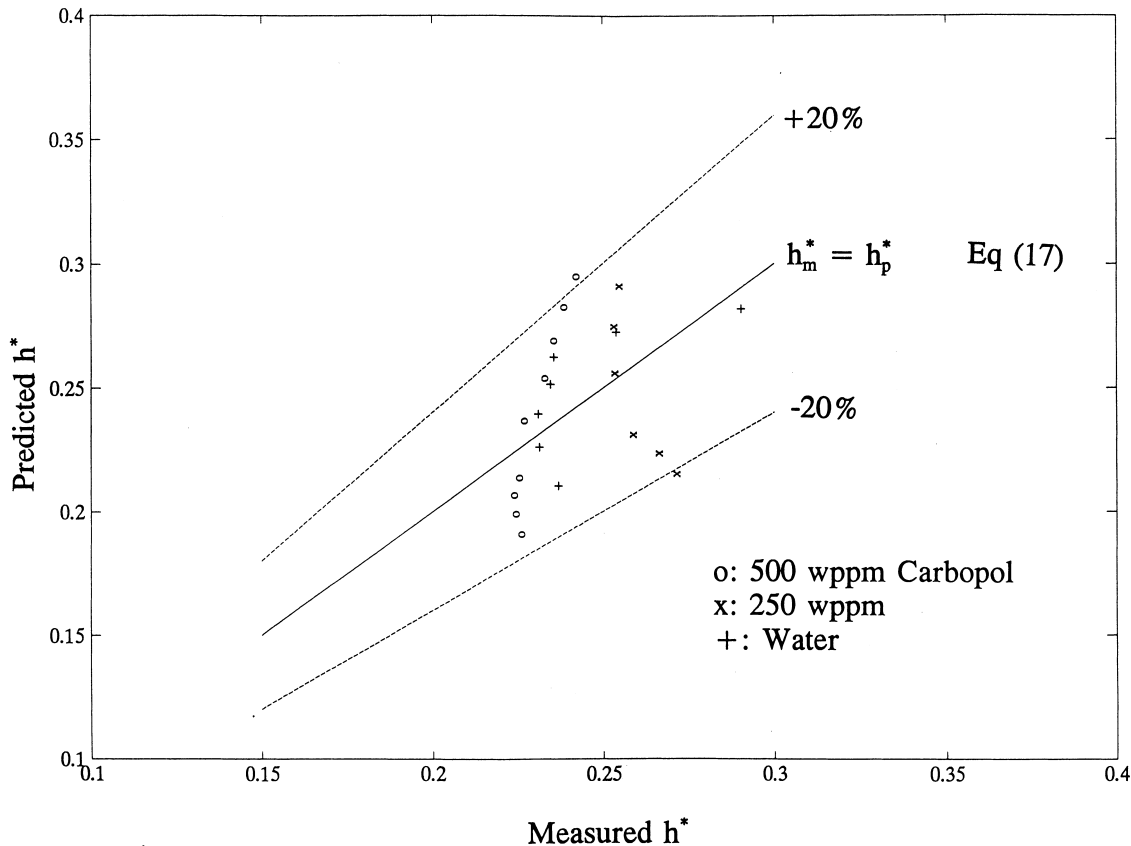


Fig. 6. Predicted versus measured dimensionless heat transfer coefficient.

with validity over a rather narrow range: $502 \leq Re_a \leq 5400$, $5 \leq Pr_a \leq 20$, and $0.7 \leq n \leq 1$. The empirical constants A , B and C , obtained by regression are 0.6, 0.4 and 0.8, respectively. Eq. (17) predicts the present laminar and turbulent h^* for power-law fluids as well as water (in the limit when $n = 1$) within $\pm 20\%$, as seen from Fig. 6. Although a polynomial is used Eq. (17a) for simplicity, it is conceivable that the function $f(n)$ can have other forms too. As mentioned in the earlier section, η , Re_a , and Pr_a were calculated at the reference $\dot{\gamma}$ (chosen from dimensional consideration) given by Eq. (7). Clearly, the (arbitrary) choice of $\dot{\gamma}$, in the absence of information on velocity distribution across the film thickness, affects the numerical values of the empirical constants in Eq. (17a). When more experimental data on power-law fluid films become available, Eq. (17) with an appropriate choice of $f(n)$ can be expected to have improved predictability and a wider range of applicability.

The estimated uncertainties, using the root-sum-square method, in the values of Re_a and h^* are 4.1% and 7.5%, respectively.

5. Conclusions

(1) Falling power-law fluid films exhibited h^* minima at two locations (on either side of that for Newtonian films).

(2) The Newtonian correlations (Eqs. (12)–(16)) underpredicted the laminar h^* and overpredicted the turbulent h^* for the aqueous solutions of Carbopol.

(3) At any fixed Re_a , the turbulent h^* for power-law fluid film is lower than that for a Newtonian film with comparable Pr_a . This heat transfer reduction increases with decreasing n value.

(4) The proposed new correlation (Eq. (17)), over a range $502 \leq Re_a \leq 5400$, $5 \leq Pr_a \leq 20$, $0.7 \leq n \leq 1$, predicts the laminar and turbulent h^* for water as well as power-law fluid films within $\pm 20\%$.

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